Network models for robo-advisory asset allocation

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FINancial TECHnologies

- Fintech denotes innovative financial services enabled by disrupting technologies (blockchain, big data analytics, artificial intelligence): crypto payments, peer to peer lending, robot advisory.
- Fintech services are competitive, but may bring higher risks: cyber risk, fraud detection, money laundering; credit risk; compliance risk, market risk. All amplified by systemic risk, due to the high interconnectdness of fintech platforms.
- Our aim is to build a risk management framework that encourages a safe digital finance, regtech and suptech.

FIN-TECH @unipv

- We have a fintech laboratory, with 5 Faculty members and 9 Phd students, in the departments of economics and of computer engineering.
- in 2019-2020 we will be coordinating a CSA European H2020 project: FINancial supervision and TECHnological compliance, aimed at training national regulators and fintechs on fintech risk manaagement.
- It will do so through the creation of an integrated European platform for fintech, with 3 research workshops, 6 R coding sessions and 72 hours of lectures x 29 countries in i) P2P lending; ii) robot advisory; iii) crypto payments.

FIN-TECH partners

UNIVERSITIES and RESEARCH CENTRES	FINTECH HUBS and ASSOCIATIONS	REGULATORS and SUPERVISORS

FinTech Project - 2018 1

Robo-Advisors: definition

"Applications that combine digital interfaces and algorithms, and can also include machine learning, in order to provide services, ranging from automated financial recommendations, to contract brokering, to portfolio management to their clients. Such advisors may be standalone firms and platforms, or can be in-house applications of financial instiutions." (FSB, 2017)

Robo-advisors: research questions

1. Can we build an automated portfolio allocation algorithm that takes multivariate network dependencies into account? CORRELATION NETWORK MODELS
2. Can a robo-advisory portfolio allocation match investors' risk preferences? NEURAL NETWORK MODELS
3. Can we build an automated portfolio allocation algorithm that includes crypto assets? PARTIAL CORRELATION MODELS

Correlation network models: background

Mantegna (1999) suggests to employ the correlation matrix to detect the clustering structure present among the assets of a financial market.

The application of hierarchical clustering techniques, such as the Minimum Spanning Tree (MST) to the correlation matrix reveals the existence of common behaviour of groups of asset returns

Minimum Spanning Trees

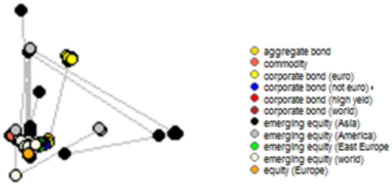
The Minimum Spanning Tree (MST) is a graph tree that allows to shrink links connecting asset returns from the $\frac{N(N-1)}{2}$ present in the empirical correlation matrix to $N-1$.

It is the minimal shortest path in terms of sum of distances between all assets.

https://en.wikipedia.org/wiki/Kruskal%27s_algorithm#/media/File:MST_kruskal_en.gif

MST: Application

Daily closing price of $N = 92$ Exchange Traded Funds (ETFs) from the Italian Stock Exchange, January 2006 to February 2018.



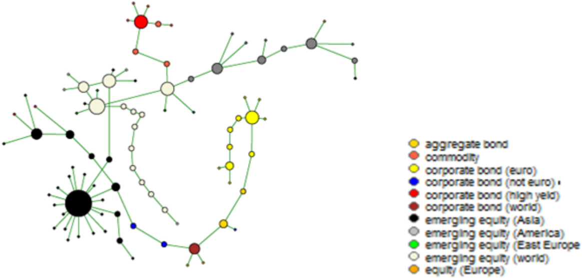
Random Matrix Theory

Plerou (2002) shows that the eigenvalues of the correlation matrix deviating from those of a random matrix convey the important information.

- RMT tests the eigenvalues $\lambda_k < \lambda_{k+1}$ of the empirical correlation matrix against the null hypothesis that they come from a same size "random" correlation matrix R
- It retains only the eigenvalues of the empirical correlation matrix that are higher than the maximum eigenvalue of R, and consequently derives a filtered correlation matrix.

MST + RMT: application

Daily closing price of $N = 92$ Exchange Traded Funds (ETFs) from the Italian Stock Exchange, January 2006 to February 2018.



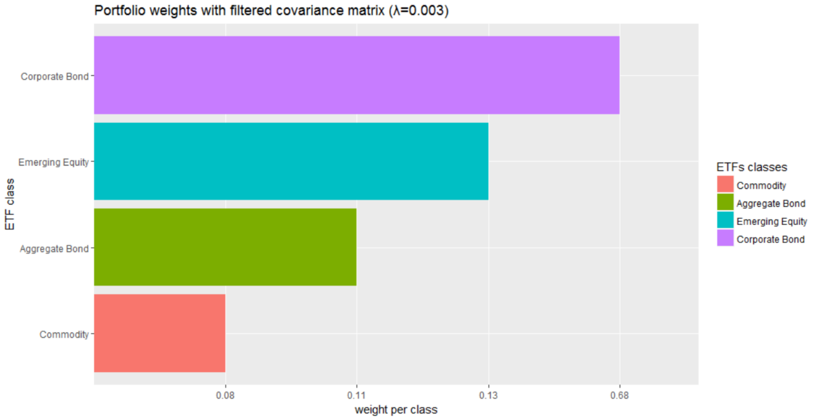
Portfolio construction

$$\min_{w} \quad \sigma_P^2 = w^T \underbrace{COV_{RMT}} w + \lambda \underbrace{\sum_{i=1}^{n} b_i w_i^2} \tag{1}$$

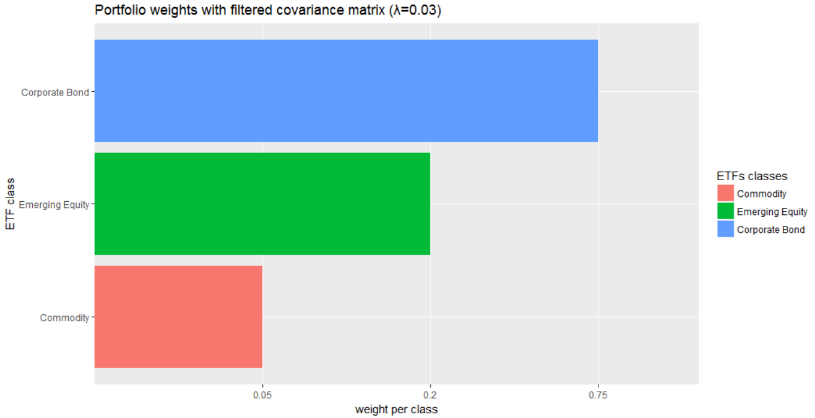
$$\text{subject to} \quad \begin{cases} \sum_{i=1}^{n} w_i = 1 \\ \mu_P \geq \frac{\sum_{i=1}^{n} \mu_i}{n} \\ wi \geq 0 \end{cases}$$

where σ_P^2 and μ_P are the portfolio variance and returns, λ expresses risk aversion and b_i is the closeness centrality of the node i.

Portfolio weights - unfiltered matrix



Portfolio weights - filtered matrix



Portfolio composition - unfiltered matrix

	id	class	weights	closeness
1	XBAG IM	Aggregate Bond	5%	0.0023
2	GAGG IM	Aggregate Bond	6%	0.0023
3	WCOA IM	Commodity	8%	0.0025
4	R1JKEX IM	Corporate Bond - Euro	68%	0.0019
5	XNIF IM	Emerging Market - Asia	9%	0.0019
6	XMBR IM	Emerging Market - America	1%	0.0028
7	RDXS IM	Emerging Market - Europe	2%	0.0031

Portfolio composition - filtered matrix

	id	class	weights	closeness
1	XDBC IM	Commodity	5%	0.0025
2	ECOEUA IM	Corporate Bond - Euro	75%	0.0016
3	XNIF IM	Emerging Market - Asia	18%	0.0019
4	XMBR IM	Emerging Market - America	2%	0.0028

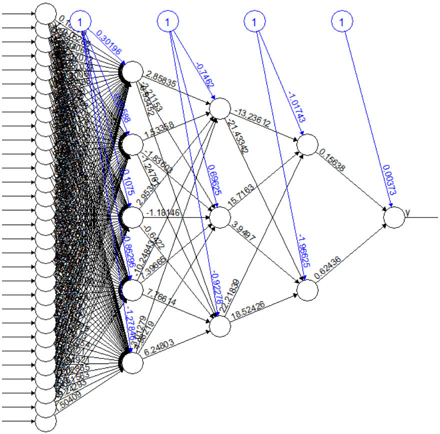
Further findings

- Portfolios based on the filtered covariance matrix are less risky in terms of portfolio variance; and contain ETFs less correlated
- Increasing risk aversion increases porfolio risk
- We are experimenting with a different measure of correlation and a time varying structure

Compliance risk - I

- We consider each ETF as a training example, with daily returns as input variables, and the cluster group label as the output variable.
- We then learn the "allocation" rule of each ETF. The more volatile days have the highest importance weights.
- We can thus use the network to predict the actual risk class of a new ETF, and for any actual investor's portfolio, and compare it with the expected risk profile.

Compliance risk - II



Network models for crypto assets

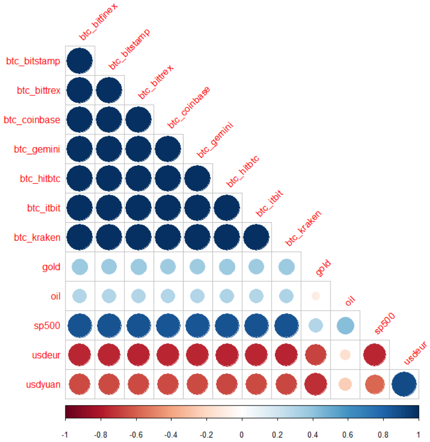
- Cryptoasset prices are very volatile and, often, very illiquid.
- A very relevant question is to understand the determinants of prices, analysing the correlation structure, among different exchange markets, and between crypto assets and regulated assets.
- We focus on bitcoins, the most liquid and well known crypto assets.

Data - crypto prices

Table: Summary statistics

Price	Mean	St. Dev.	Min	Max
Bitfinex Bitcoin	3899.56	4274.46	435.61	19187.12
Coinbase Bitcoin	3919.05	4318.98	438.38	19650.01
Bitstamp Bitcoin	3899.04	4286.02	439.62	19187.78
HitBtc Bitcoin	3916.19	4297.17	436.36	19095.30
Gemini Bitcoin	3910.38	4306.36	437.57	19475.90
ItBit Bitcoin	3907.13	4300.32	438.61	19357.97
Kraken Bitcoin	3890.18	4272.55	433.50	19356.91
Bittrex Bitcoin	3893.83	4269.89	421.11	19261.10
Gold	1275.57	52.34	1128.42	1366.38
Oil	48.67	3.16	39.51	54.45
SP500	2414.78	212.308	2000.54	2872.87
USDYuan	6.67	0.19	6.26	6.96
USDEur	0.88	0.04	0.80	0.96

Correlation structure



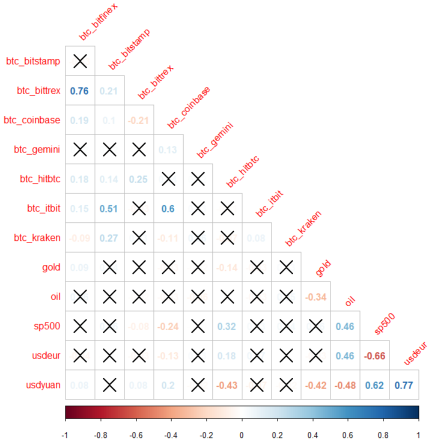
Partial Correlations

- From the inverse of the covariance matrix (A^{-1}, elements σ^{mn}), partial correlations can be derived:

$$\rho_{mn|rest} = \frac{-\sigma^{mn}}{\sqrt{\sigma^{mm}\sigma^{nn}}}.$$

- They represent correlations between two prices, conditional on the remaining prices of the system (*rest*).

Partial correlation structure



Partial Correlation network

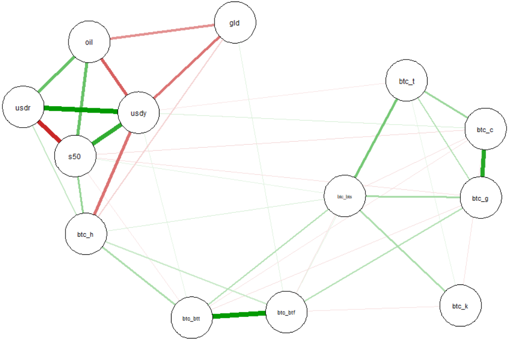


Figure: Partial correlation network model between prices.

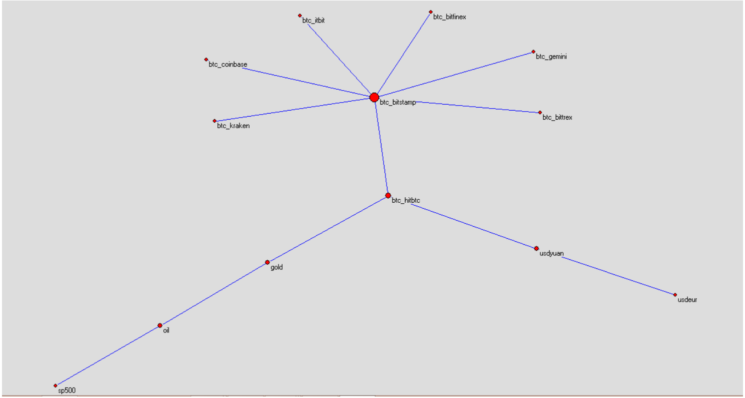


Figure: Minimum spanning tree between crypto prices, after random matrix theory filtering.

Predictive modelling - I

We assume a CoRisk model, as in Giudici and Parisi (QF 2017, RISKS 2018).
Let i be a market price (crypto, forex, share). We assume prices y_t^i follow a structural VAR process:

$$y_t^i = \sum_{p=1}^{T} \beta_p^i y_{t-p}^i + \sum_{j \neq i} \gamma^j y_t^j + \varepsilon_t^i.$$

or

$$Y_t = \sum_{p=1}^{T} A_p Y_{t-p} + B_0 Y_t + u_t,$$

where B_0 has null diagonal elements.

Predictive modelling - II

We can transform the VAR to its reduced form:

$$Y_t = \Gamma_1 Y_{t-1} + \ldots + \Gamma_p Y_{t-p} + u'_t,$$

$$\begin{cases} \Gamma_1 = (\mathbb{I} - B_0)^{-1} A_1, \\ \ldots \\ \Gamma_p = (\mathbb{I} - B_0)^{-1} A_p, \\ u'_t = (\mathbb{I} - B_0)^{-1} u_t. \end{cases}$$

Need to estimate B_0 to derive $A_1, \ldots, A_p$. Note that $u'_t = B_0 u'_t + u_t$:

$$\begin{cases} [u'_t]^i = \sum_{j \neq i} b_0^j [u'_t]^j + [u_t]^i, \\ \gamma^{ij} = \sqrt{b_0^j b_0^i} = corr(y_i, y_j | rest) \end{cases}$$

Results: predictive performance

Table: Predictive performance summary

Prices	RMSE full	RMSE autoreg
Bitfinex Bitcoin	267.37	293.49
Coinbase Bitcoin	550.10	579.98
Bitstamp Bitcoin	379.71	397.25
HitBtc Bitcoin	290.63	342.50
Gemini Bitcoin	792.22	786.58
ItBit Bitcoin	331.62	455.45
Kraken Bitcoin	718.51	676.35
Bittrex Bitcoin	288.84	305.51
Oil	0.55	0.58
SP500	5.90	6.35
USDYuan	0.02	0.02
USDEur	0.002	0.003
Gold	6.74	7.19